

## MINIMUM INSERTION LOSS MICROWAVE FILTERS

J. J. Taub and H. J. Hindin

Airborne Instruments Laboratory      Deer Park, N. Y.

Equal-element band-pass filters provide the lowest midband insertion loss possible with a given number of resonators and a specified rejection bandwidth or skirt response.<sup>1</sup> Losses can often be reduced by 1 to 2 db. A technique applicable to narrow-band equal-element filters has been developed that holds for both large and small dissipation factors.<sup>2</sup> Using this technique, circuit constants can be determined for filters having from two to eight coupled resonators. Formulas are given for lumped-constant and strip-transmission-line circuits.

This paper shows the application of this technique to the design of microwave filters (waveguide, coaxial line, and strip transmission line). SWR and time delay are derived and are superior to Butterworth and Tchebycheff filters of comparable skirt response. Experimental insertion loss and SWR for an experimental waveguide filter agrees closely with the theory.

The theoretical analysis of equal-element filters was performed using equivalent circuits. Figure 1A shows a band-pass filter consisting of identical series-resonant circuits with shunt-reac-

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1 S. B. Cohn, "Dissipation Loss in Multiple-Coupled Resonator Filters," IRE Proceedings, August 1959.

2 J. J. Taub, "Design of Minimum-Loss Band-Pass Filters," Technical Memorandum No. 38, Airborne Instruments Laboratory, May 1962.

tive coupling. The equal-element filter has the same coupling coefficient ( $K$ ) between each resonator pair, and  $W_0 LK = R$  where  $R$  is the source and load resistance. Figure 1B shows a normalized equivalent circuit obtained by choosing  $R = 1$  ohm.

The properties of this network are a function of a single complex variable:  $X = u + jv$ . The parameter  $u = v_r/R = 1/KQ_u$  is the normalized dissipation factor and  $v = 1/K (f/f_0 - f_0/f) = F/K$  is the normalized frequency variable.

Figure 1C shows this network represented by a cascade of identical sections ( $n$ ) and a single reactive tee circuit. Insertion loss, SWR, and time delay as a function of  $u$  and  $v$  are obtained from the ABCD matrix.<sup>2,3</sup> These quantities were obtained with a digital computer for  $n = 2$  through  $n = 8$  resonators. The results are for  $n = 4$ . Figures 2 and 3 show SWR and time delay as a function of frequency.

The filter has been designed with the aid of insertion loss curves.<sup>2</sup> Specifying a maximum midband insertion loss,  $u$  is determined by noting which  $u$  curve intersects the desired  $L$  at  $v = 0$ . If an off-resonance rejection is then specified,  $v$  can be determined by noting which  $v$  value for the proper  $u$  curve corresponds to the desired rejection for a frequency deviation  $F$ . The required  $K$  and  $Q_u$  can then be determined. It is a simple matter to convert to the required waveguide, coaxial, or strip transmission line geometry

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<sup>3</sup> L. Storch, "The Transmission Matrix of N Alike Cascaded Networks," AIEE Transactions, January 1955.

Figure 2 shows that the midband SWR should be near unity for both lossy and lossless filters ( $u = 0$ ). The effect of loss is to smooth out the variations in the pass band, which is often beneficial. The insertion loss of a Butterworth filter having losses corresponding to  $u = 0.2$  or  $0.3$  would be appreciably higher (at least 1.5 at midband).

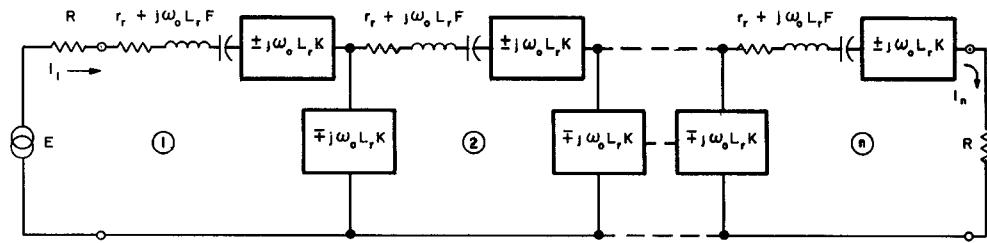
The time delay in Figure 3 is flat within 10 percent over at least half of the pass band for the  $u$  values plotted. Also, the increased dissipation lessens the time-delay variation over the entire pass band. These results suggest the possibility of obtaining flatter delay in filter networks by designing them with equal elements and deliberately using high  $u$  values (low  $Q$  resonators).

An equal-element filter was constructed to operate with a center frequency near 9 Gc, an 8-Mc 3-db bandwidth, and a 17-Mc bandwidth for points 30 db below the peak of the response. This filter used four silver-plated rectangular waveguide resonators with circular irises for coupling elements. The  $Q_u$ 's of the resonators were about 4600. The experimental and theoretical results are compared in Figure 4.

The theoretical curve corresponds to  $u = 0.32$ . Agreement is within 0.2 db in the pass band and 1 db in the reject band. The midband insertion loss is 5.5 db, which is 2.5 db lower than a filter designed for an exact maximally flat response.<sup>4</sup>

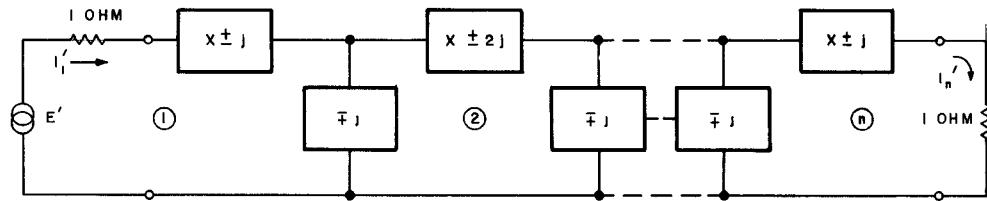
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<sup>4</sup> E. G. Fubini and E. A. Guillemin, "Minimum Insertion Loss Filters," IRE Proceedings, January 1959.



FOR EQUAL-ELEMENT RESPONSE,  $\omega_0 L K = R$   
 THE  $\mp j\omega_0 L K$  REFER TO CAPACITIVE AND INDUCTIVE COUPLING RESPECTIVELY

A. MESH EQUIVALENT CIRCUIT OF A DIRECT-COUPLED BAND-PASS FILTER

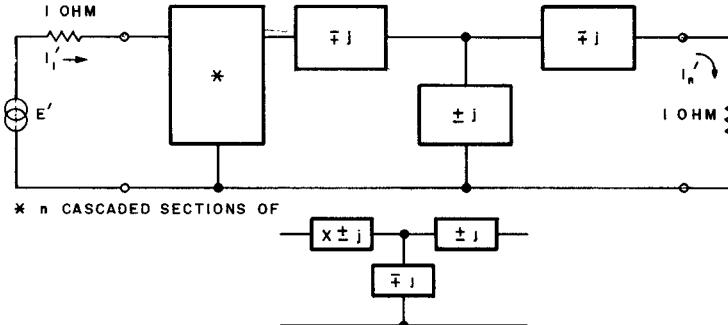


WHERE  $X = u + jv$

$$u = \frac{r_r}{R}$$

$$v = \frac{F}{K}$$

B. MESH EQUIVALENT CIRCUIT NORMALIZED TO 1-OHM LOAD RESISTANCES



C. NORMALIZED CIRCUIT REPRESENTED AS AN ITERATIVE NETWORK

Fig. 1. Mesh equivalent circuits of equal-element filters.

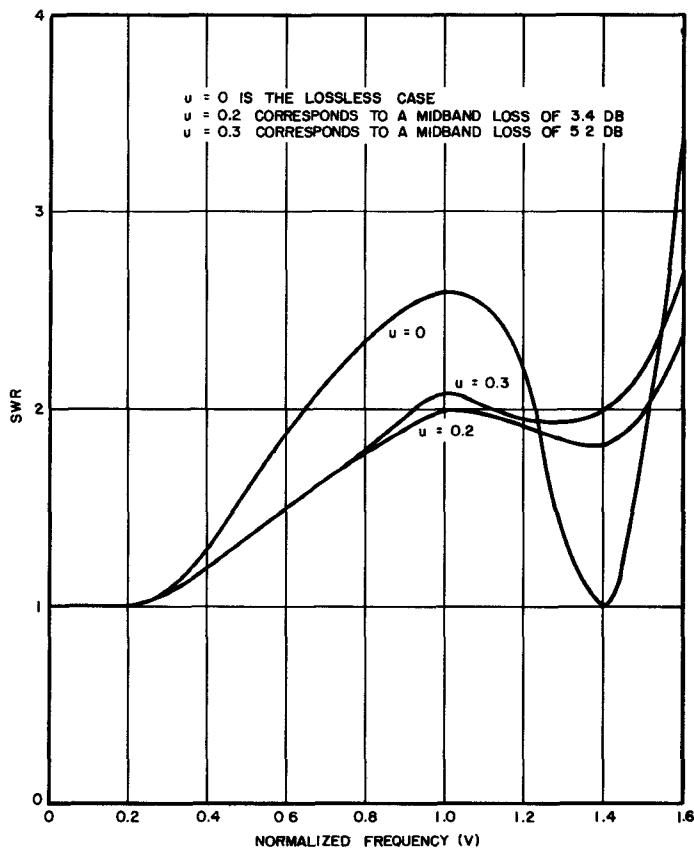


Fig. 2. SWR versus frequency for four-resonator equal-element filters.

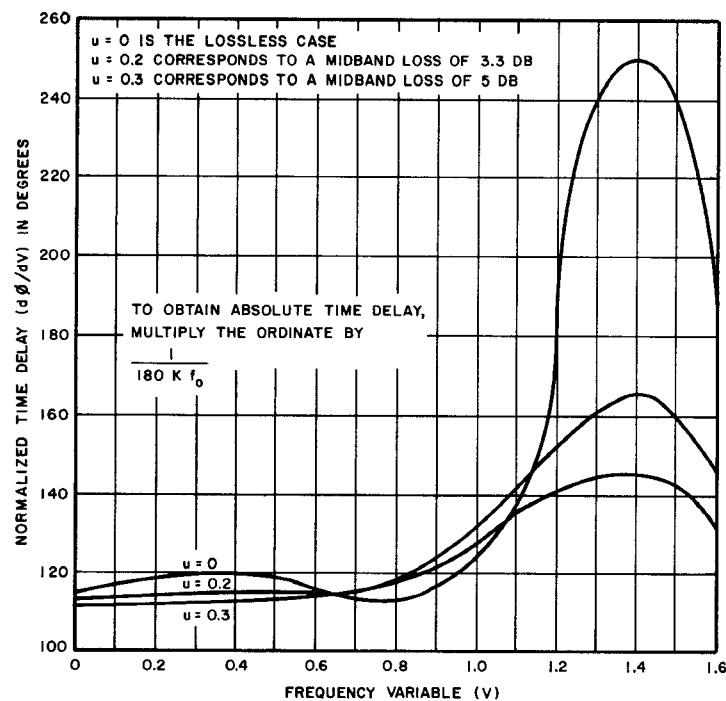


Fig. 3. Time delay versus frequency for four-resonator equal-element filters.

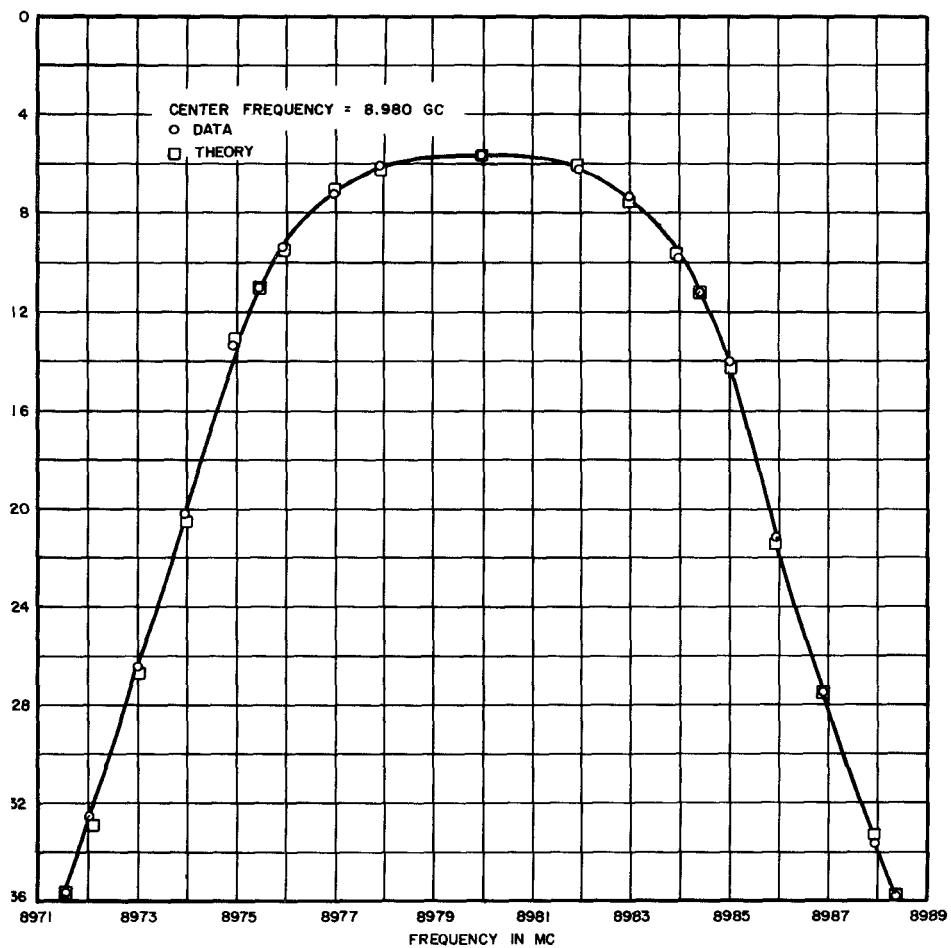


Fig. 4. Insertion loss versus frequency for four-resonator equal-element filters.

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# NOTES

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I-T-E CIRCUIT BREAKER COMPANY  
SPECIAL PRODUCTS DIVISION  
601 E. ERIE AVENUE  
PHILADELPHIA 34, PENNSYLVANIA

"Designers and Manufacturers of antennas, towers, rotary joints, pedestals, waveguide and microwave components for radar and communications."